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Coherent transition radiation at submillimeter and millimeter wavelengths

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A spectrum of coherent transition radiation (TR) emitted from 150-MeV electron bunches from a linear accelerator has been observed in the wavelength range from 0.6 to 5 mm. The intensity at a wavelength of 4 mm is enhanced by a factor of about 6×10^5 in comparison with that of ordinary TR. The factor is about a half of the number of electrons in a bunch. The intensity shows nearly a quadratic dependence on the electron-beam current. The interference between TR from adjacent bunches has been observed. The electron distribution in a bunch has been derived from the observed spectrum. It has a full width at half maximum of 0.56 mm and the shape is similar to a Gaussian function.

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I. INTRODUCTION

Transition radiation (TR) is emitted when electrons pass across a boundary between two media. In the case of bunched electrons, enhancement of the intensity of TR due to a coherence effect is expected in the region where the wavelength is longer than the longitudinal length of a bunch.

In the far-infrared region, coherent TR has been observed, which is emitted from relativistic electrons using linear accelerators [1–3]. Shibata *et al.* [1,3], observed the spectrum of coherent TR at the millimeter wavelengths using an *L*-band linac. Happek, Sievers, and Blum [2] reported that the intensity of TR was much larger than that of Cherenkov radiation in SF_6 gas in the far-infrared region.

The aim of this paper is to clarify properties of coherent TR emitted from periodic bunches using an *S*-band linac in the long-wavelength region. We report the observed spectrum over a wide wavelength range from 0.6 to 5 mm, the dependence of the intensity on the electron-beam current, and the coherent property of TR from successive bunches. Furthermore, the observed spectrum is analyzed to derive the electron distribution in a bunch. It is shown that the spectrum of coherent TR is useful as a high-resolution analyzer of the bunch shape. Hitherto, coherence effects of the bunched electrons in the far-infrared region have been investigated for synchrotron radiation [4–8].

In our experiment, TR was emitted from relativistic electrons across a system consisting of a metallic plate and a metallic mirror, as described below in Sec. II. The intensity of TR from such a system in vacuum, which is expressed as the number of photons per unit wavelength ($d\lambda$) and unit solid angle ($d\Omega$), is given by [9]

$$P=2P_f\{1-\cos(L/Z)\}\ , \tag{1}$$

$$P_f=\frac{\alpha\beta^2\sin^2\theta\cos^2\theta}{\pi^2\lambda(1-\beta^2\cos^2\theta)^2}|\zeta|^2\ , \tag{2}$$

$$\zeta=\frac{(\epsilon-1)[1-\beta^2-\beta(\epsilon-\sin^2\theta)^{1/2}]}{[\epsilon\cos\theta+(\epsilon-\sin^2\theta)^{1/2}][1-\beta(\epsilon-\sin^2\theta)^{1/2}]}\ , \tag{3}$$

$$Z=\frac{\beta\lambda}{2\pi(1-\beta\cos\theta)}\ , \tag{4}$$

where α, β, ϵ, and θ are, respectively, the fine-structure constant, the ratio of the velocity of an electron to that of light, the dielectric constant, and the angle between the direction of the observation point and that of the electron. The quantity L is called the emission length, which is the length between the metallic plate and the mirror. The quantity Z is called the formation length. The value of $|\zeta|^2$ in Eq. (2) is close to unity, since the dielectric constant of the metal is much larger than unity in the far-infrared region ($|\epsilon|\gg1$).

In the present experiment, the trajectory of the electron beam which corresponded to the emission length was not in vacuum but in air at the atmospheric pressure. In a dielectric medium such as air, Cherenkov radiation is also emitted when the velocity of an electron exceeds that of light in the medium. However, since the emission length is small and the refractive index of air is close to unity, the intensity of the Cherenkov radiation is negligible [10].

In the case of the radiation from a train of bunched electrons, the intensity of coherent TR is given by [3]

$$P(\lambda)=P_0(\lambda)N_e\{1+N_ef(\lambda)\}G(\lambda)\ , \tag{5}$$

$$G(\lambda)=\left\{\frac{\sin(\pi L_BN_B/\lambda)}{\sin(\pi L_B/\lambda)}\right\}^2\ , \tag{6}$$

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$$f(\lambda)=\left|\int_{-\infty}^{\infty} S(r)\exp\left[\frac{i2\pi r}{\lambda}\right]dr\right|^2 , \qquad (7)$$

where N_e, N_B, and L_B denote the number of electrons in a bunch, the number of bunches, and a distance between successive bunches, respectively. The quantity $G(\lambda)$ is reduced to N_B when the resolution of the spectrometer is lower than $1/L_B$. In Eq. (7), $f(\lambda)$ is called the bunch form factor and is expressed by the Fourier transform of the density distribution function $S(r)$ of an electron in a bunch [11].

II. EXPERIMENTAL PROCEDURES

The arrangement of the experiment is shown in Fig. 1. Electrons were accelerated to 150 MeV by the Tohoku 300-MeV linac with an rf frequency of 2856 MHz. The energy spread, the duration of a burst, and its repetition were 0.2%, 2 μs, and 300 pulses/s. The electron beam emerged from vacuum into air through an aluminum window *W* in Fig. 1. The beam position and the intensity of the current were monitored by a fluorescent screen *S* and a secondary emission monitor (SEM). The screen *S* was an aluminum plate which was 2 mm thick. The average beam current was typically 0.3 μA during the experiment, which corresponded to 1.1×10^6 electrons in a bunch. Transition radiation emitted from the screen *S* was reflected at right angles by a flat aluminum foil M_1 whose size was 110 mm in diameter. In addition, TR was emitted from the foil M_1 at right angles to the direction of the electron trajectory. The emission length of TR between *S* and M_1 in Fig. 1 was 25 cm. The acceptance angle of a spherical mirror M_2 was 57 mrad. The radiation was led to a polarizing interferometer of the Martin-Puplett type and detected by two liquid-helium-cooled Si bolometers. One was used to observe the interferogram and the other to monitor the fluctuation of the TR intensity caused by the beam instability. Both bolometers had long-wavelength-pass filters with a cutoff wavelength of 286 μm. The sensitivity of the measuring system was calibrated by blackbody radiation emitted from a graphite cavity at a temperature of 1200 K [7]. The experimental accuracy of the absolute intensity of the observed TR was estimated to be within a factor of 1.4.

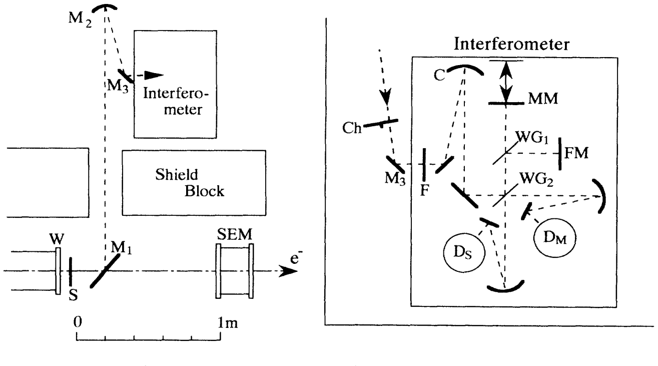

FIG. 1. The arrangement of the experiment. Keys are *W*, aluminum window (thickness of 2 mm); *S*, aluminum screen coated ZnS (thickness of 0.5 mm); M_1, flat aluminum foil; M_2 spherical mirror; M_3, plane mirror; Ch, chopper; *F*, filter; *C*, collimator; WG_1,WG_2, wire grids; FM, fixed mirror; MM, movable mirror; D_S,D_M, helium-cooled Si bolometers for measuring interferogram and for monitoring intensity; SEM, secondary emission monitor; and e^-, electron beam.

III. RESULTS AND DISCUSSION

A. Spectrum

The spectrum of coherent TR was obtained by the Fourier transform of the interferogram. An observed spectrum at the average beam current of 0.3 μA is shown in Fig. 2 by the solid curve. The maximum optical path difference of the interferogram was 5 mm and the resolution was about 2 cm^{-1} (60 GHz). The spectrum has a broad peak around $\lambda\sim1.5$ mm and rapidly decreases on the short-wavelength side.

The intensity of incoherent TR at the average beam current of 0.3 μA is also shown in Fig. 2 by the broken line, which is calculated for the geometry of the experiment. In comparison with the calculated intensity of incoherent TR, the observed intensity of coherent TR at $\lambda\sim4$ mm is enhanced by a factor of about 6×10^5. The factor is close to the number of electrons in a bunch, 1.1×10^6. This result indicates that the enhancement of the intensity is due to the coherence effect.

B. Dependence of intensity on beam current

The dependence of the intensity of TR detected by the Si bolometer, D_S in Fig. 1, on the electron-beam current was observed and the result is shown in Fig. 3 by the circles. The current is proportional to the numbers of elec-

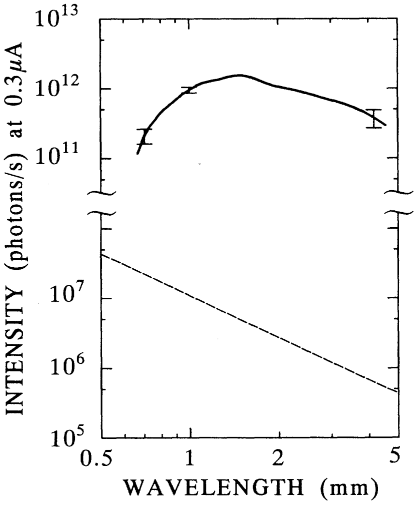

FIG. 2. The observed spectrum of coherent transition radiation. The resolution of the spectrum is about 2 cm^{-1} (60 GHz). The intensity is shown in units of the photon numbers per second per bandwidth of 1%. The vertical bars show the errors of experiment. The broken curve shows the calculated intensity of ordinary transition radiation.

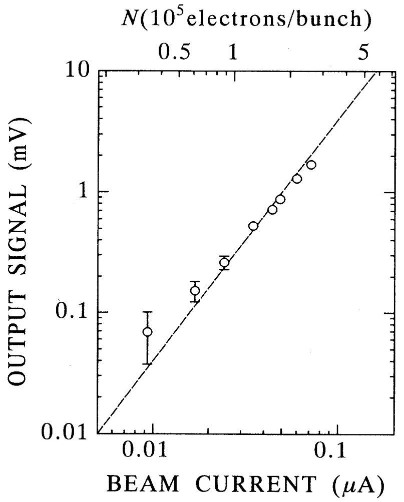

FIG. 3. The relation between the intensity and the average beam current. The number of electrons in a bunch is also shown on the upper axis instead of the beam current. The intensity shows the output signal of the detector D_S in Fig. 1. The broken line shows the quadratic dependence on the beam current.

trons in a bunch.

In the case of complete coherent radiation, the intensity should be proportional to the square of the beam current. The broken line in Fig. 3 shows the quadratic dependence, and the observed data are on the line almost within the error of experiment. By means of the method of least squares, however, the gradient of the observed data is obtained as 1.67 ± 0.04. The reason for the deviation of this value from 2 is not clear at present, but it is shown that the intensity is nearly proportional to the square of the beam current.

C. Coherent property

Transition radiation was emitted from periodic bunches accelerated with an rf frequency of 2856 MHz, and the distance between successive bunches was 105 mm. Hence, the interference of TR from adjacent bunches should be observed when the optical path difference of the interferometer is 105 mm.

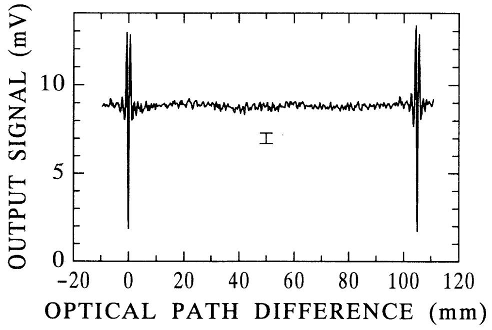

FIG. 4. The observed interferogram of coherent transition radiation. The sampling interval was 0.1 mm in the optical path difference. The vertical bar shows the error of experiment, which is about 0.6 mV.

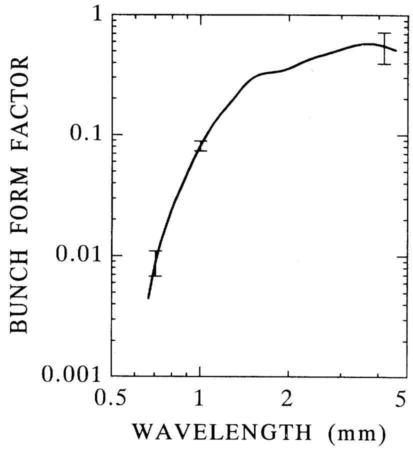

FIG. 5. The bunch form factor calculated from the observed spectrum of coherent transition radiation.

The observed interferogram, of which the maximum path difference is 110 mm, is shown in Fig. 4. The interferogram has a sharp pattern with a deep minimum at an optical path difference of 0 mm, and the same pattern is repeated at 105 mm. This result confirms that TR from separate bunches are coherent. In the case of coherent synchrotron radiation, the interference between the radiation from separate bunches has already been reported [5].

D. Bunch shape

The bunch form factor is derived from the observed spectrum using Eq. (5). The result is shown in Fig. 5. The density distribution function of an electron in a bunch is calculated from the bunch form factor by the in-

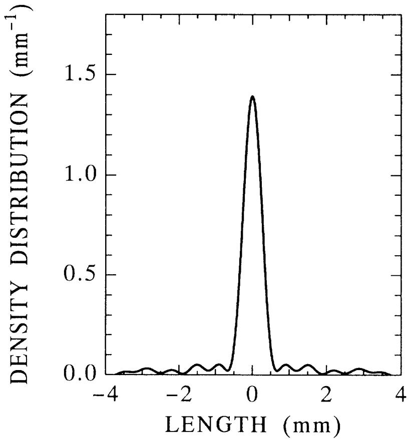

FIG. 6. The density distribution of an electron in a bunch calculated from the bunch form factor. Its shape is similar to the Gaussian function with the full width at half maximum of 0.56 mm.

verse Fourier transform of Eq. (7), and the result is shown in Fig. 6. Its shape is similar to the Gaussian function with the full width at half maximum of 0.56 mm.

The longitudinal length of a bunch in the linac has been estimated at 1.7 mm from the phase distribution of a bunch [12]. The observed bunch length is found to be about one-third the estimated one.

Using the same linac, Ishi *et al.* [7], obtained the longitudinal bunch length of 0.25 mm from the observed spectrum of coherent synchrotron radiation. They suggested that the bunch had a local concentration of electrons of the size of 0.25 mm, on the bases of the simulation of motion of electrons in the accelerator. The sharp concentration of electrons in a bunch is in qualitative agreement with the result obtained in this paper. The difference between the longitudinal bunch lengths obtained from coherent synchrotron radiation and from coherent transition radiation is considered to be mainly attributed to the operation of the linear accelerator. From our experience, it seems that the longitudinal bunch length changes from time to time due to uncontrollable shifts of some parameters of operation.

IV. CONCLUSION

The present experiment has shown that coherent TR is useful not only as the intense light source in the far-infrared region but also as an analyzer of the bunch form. A conventional method of observing the bunch form, e.g., using a streak camera, has the resolution of a few millimeters at most. On the other hand, the spectrum of coherent TR can easily be observed in the shorter-wavelength region than 1 mm. We may conclude that the observation of the spectrum of coherent TR is a useful means of the high-resolution measurement of the bunch form.

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